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HEAT TRANSFER IN A PISTON–CYLINDER SYSTEM

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NOMENCLATURE

- A, area;
- a, thermal diffusivity;
- c_m, piston velocity;
- c_p, specific heat;
- D, dimensionless quantity, defined by (8);
- l, length;
- P, motor input power;
- p, pressure;
- Q, heat flow rate;
- q, heat flux density;
- r, ratio l_s/l_{th};
- T, temperature;
- t, time;
- u, gas velocity;
- V, gas volume;
- x, coordinate perpendicular to the boundary layer;
- y, $\dfrac{\lambda_o T_w \delta_v^2 (\gamma - 1)^2 D}{2\sqrt{(2)}\, l_{th}} \cdot \dfrac{\cos(\varphi)}{\cos(\pi/4)}$;

Greek symbols

- α, heat transfer coefficient;
- γ, ratio of specific heats;
- δ, modulation depth;
- λ, thermal conductivity;
- ν, kinematic viscosity;
- ω, angular frequency;
- ρ, gas density;
- τ, transformed time;
- Φ, T/T_b;
- φ, phase angle;
- ξ, transformed coordinate.

Subscripts

- A, surface A;
- a, area;
- B, surface B;
- b, bulk;
- f, flow;
- g, gas;
- L, loss;
- s, boundary layer;
- p, pressure;
- T, temperature;
- th, thermal;
- V, volume;
- w, wall;
- o, normal.

INTRODUCTION

IN THIS note some preliminary results on measurements and calculations are presented regarding gas–wall heat transfer in a cylinder in which a piston is periodically moved. Such a system can be considered as representative of a gas compressor or a Stirling cycle machine. The method of determining the heat transfer coefficient described in this note differs from those given in the literature [1, 2] in that periodicity of the piston motion is considered.

Particular attention is paid to the influence of the variation of the heat-transferring surface area. It is shown that this variation may have considerable effect on the heat transfer properties of the piston–cylinder system.

THEORETICAL DESCRIPTION OF THE SYSTEM

The system under consideration is shown schematically in Fig. 1. A piston is moved sinusoidally in a closed cylinder, whose wall surface is divided into three parts, labeled A, B and C. The heat flux to parts A and B is measured separately, part C is insulated. The piston is driven by an electrodynamic linear motor, whose operating frequency equals the resonant frequency of the gas–spring–mass system, consisting of the mass of the moving part of the system and the gas on both sides of the piston [3].

The equations describing the properties of the time-dependent boundary layer of the gas, in turn determining the heat transport, are the equations of continuity and energy conservation [1].

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{1}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) = \frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right). \tag{2}$$

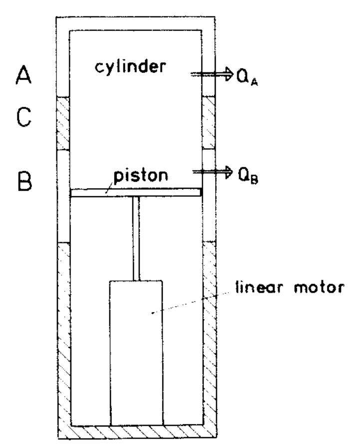

FIG. 1. Schematic set up. The piston movement covers part B of the cylinder wall only. Q_A and Q_B are defined in expressions (9) and (10) indicating that Q_B also includes the instantaneous heat flow below the piston position as drawn in the figure.

In this description viscous dissipation is neglected, while the geometry of the boundary layer is taken to be one-dimensional. (Note that the x-coordinate is taken perpendicular to the boundary layer.) This is justified by the fact that the heat transfer due to gas flow parallel to the cylinder walls is lower than that due to temperature fluctuations in the bulk of the gas (see also the experimental section of this paper). Introducing the new variables Φ, ξ, and τ, defined by

$$\frac{\partial \xi}{\partial x} = \frac{\rho}{\rho o}, \quad -\frac{\partial \xi}{\partial t} = u\rho/\rho_o, \quad \frac{d\tau}{dt} = \frac{p}{p_o},$$

$$\Phi = T/T_b,$$

we obtain the following easily solvable equation

$$\frac{\partial \Phi}{\partial \tau} = a_o \frac{\partial^2 \Phi}{\partial \xi^2}. \quad (3)$$

In the above expressions T is the gas temperature, and T_b is the gas temperature outside the boundary layer, while p is the uniform pressure of the gas. It was assumed that the thermal conductivity of the gas is proportional to the temperature T, which is valid for helium and hydrogen in the temperature range of interest. The quantities labeled by o refer to values of the parameters at $T_o = 296$ K, while $a_o = \lambda_o/\rho_o c_p$, the thermal diffusivity.

In the system studied (see Fig. 1) the gas volume is given by

$$V(t) = V_o[1 + \delta_v \cos(\omega t)], \quad (4)$$

with $\delta_v \ll 1$ (see Table 1 for the values of δ_v used in the experiments). The volume variation gives rise to a relative pressure modulation of $\delta_p = \gamma\delta_v$, and a relative temperature modulation of $\delta_T = (\gamma - 1)\delta_v$.

It has been shown by a numerical solution of the equation of the energy conservation of the whole system, that both the use of γ and the use of a phase angle between the variations of V and T equal to π is justified in the present case if $\delta_v \ll 1$. In other words: the actual heat transfer has only a limited effect on the phase angle and the state changes can be described mathematically as isentropic. Now for $\delta_v \ll 1$ the bulk gas temperature T_b can be written as

$$T_b(t) = T_g[1 - \delta_T \cos(\omega t)]. \quad (5)$$

The boundary conditions of equation (3) are given by

$$\Phi(l_s, \tau) = 1 \quad \text{and} \quad \Phi(0, \tau) = T_w/T_b, \quad (6)$$

which means that the temperature of the gas is assumed to be uniform by turbulent mixing for $\xi > l_s$, the boundary layer thickness, while the gas temperature at $\xi = 0$ equals the cylinder wall temperature. References [4–6] show that l_s can be taken constant.

Solving equation (3), using the boundary conditions (6), yields the following expression for the heat flux at the wall ($\xi = 0$, or $x = 0$),

$$q(t) = \frac{\lambda_o\left(\dfrac{\delta_T T_w}{l_{th}} D \cos(\omega t + \varphi) + \dfrac{T_g - T_w}{l_s}\right)}{1 + \delta_T \cos(\omega t)}, \quad (7)$$

where l_{th} is the penetration depth for temperature waves $\sqrt{(2a_o/\omega)}$,

$$\varphi = \pi/4 - \arctan\left(\frac{\sin(2r)}{\sinh(2r)}\right),$$

with $r = l_s/l_{th}$, and D is given by

$$D = \sqrt{2}\frac{[\sinh^2(2r) + \sin^2(2r)]^{1/2}}{\cosh(2r) - \cos(2r)}. \quad (8)$$

It is easy to show that for $r \gg 1$, when $\varphi = \pi/4$ and $D = \sqrt{2}$, the expression, given by Pfriem [4] for heat transfer in a turbulence free, periodically compressed gas is obtained from expression (7). The expression (7), however, includes the effect of the finite dimension of the boundary layer on the heat transfer to the walls in a periodically compressed gas. Using expression (7) for the heat flux, the heat flow through the surfaces A and B can be determined, taking into account that the surface area A_A of A is constant and that the surface area of B is time-dependent, $A_B = A_{Bo}[1 + \delta_a \cos(\omega t)]$, where δ_a is proportional to δ_v, the proportionality-constant being dependent on the geometry of the system.

The heat flows through A and B can be calculated by integration of expression (7) multiplied by the corresponding (instantaneous) surface areas over one period of the piston motion.

$$Q_A = \oint A_A q(t) dt$$

is found to be equal to

$$Q_A = yA_A + \frac{yA_A\delta_a/\delta_v}{(1 + A_A/A_{Bo})(\gamma - 1)} + \frac{\lambda_o(T_B - T_B)A_A}{(1 + A_A/A_{Bo})l_s}, \quad (9)$$

while

$$Q_B = \oint A_B(t) q(t) dt$$

is found to be

$$Q_B = yA_{Bo} - \frac{yA_{Bo}\delta_a/\delta_v}{(1 + A_{Bo}/A_A)(\gamma - 1)} - \frac{\lambda_o(T_B - T_A)A_{Bo}}{(1 + A_{Bo}/A_A)l_s}. \quad (10)$$

In these expressions T_A and T_B are the wall temperatures of the surfaces A and B, respectively, y is defined

$$y = \frac{\lambda_o T_w \delta_v^2(\gamma - 1)^2 D}{2\sqrt{2}\, l_{th}} \cdot \frac{\cos(\varphi)}{\cos(\pi/4)}. \quad (11)$$

The expressions (9) and (10) have been derived under the assumption that the total power input into the gas volume is not affected by heat transfer from A to B. By numerical solution of the energy conservation equation of the whole system this assumption has been shown to be correct in the present discussion, where δ_v is taken to be small as compared to one.

The first term on the right-hand side of (9) and (10) is identical with that presented by Pfriem in his analysis of heat transfer [4], with $r \gg 1$. The second term is due to the variable heat transferring surface in the system, while the third term represents the heat transfer from one part of the cylinder to the other part if these parts have different wall temperatures. In many cases the latter term can be taken zero. It should be noted that even if $T_B = T_A$ the heat flow to the constant area surface A is higher by a factor

$$1 + \frac{\delta_a/\delta_v}{(1 + A_A/A_{Bo})(\gamma - 1)}, \quad (12)$$

as compared with the expression given by Pfriem, integrated over one period. This is due to the heat transfer from the surface B to the gas during that part of the period where A_B is large and $T_b < T_B$. In fact this means that heat is transferred from B to A via the periodically compressed and expanded gas. It will be clear that this also holds if $T_B \neq T_A$.

EXPERIMENTAL RESULTS

In the experiments Q_A was measured calorimetrically as a function of δ_V. The heat flow through B was also measured but its value is not exactly that of Q_B as derived above, due to the fact that in the measured Q_B the friction losses of the piston, the ohmic losses of the linear motor and the $\oint p dV$ losses of the part of the compressor below the piston (see Fig. 1) were included as well.

Table 1. Some results obtained for He (53 bar)

Piston stroke	δ_V	Q_A	Q_L	Q_B	P
$2.0 \cdot 10^{-2}$ m	0.056	9.4 W	1.4 W	15.8 W	24.4 W
$2.5 \cdot 10^{-2}$ m	0.070	14.8 W	2.8 W	24.5 W	39.6 W
$3.0 \cdot 10^{-2}$ m	0.084	19.3 W	3.9 W	35.9 W	55.0 W

The geometric parameters in the experiments were $V_o = 1.09\ 10^{-4}\ \mathrm{m}^3$, $A_A = 61.58\ 10^{-4}\ \mathrm{m}^2$, $A_{Bo} = 45.74\ 10^{-4}\ \mathrm{m}^2$, cylinder radius $= 0.014$ m and $\delta_a/\delta_T = 3.42$, while $\omega = 314\ \mathrm{s}^{-1}$. This means that the values of Q_A calculated with expression (12) are 3.31 times the values that would have been found if Pfriem's expression had been used [first term of expression (9)].

In Table 1 Q_A and Q_B are the measured values defined above, Q_L is the ohmic loss term of the linear motor, P is the linear motor input and δ_V is the relative volume modulation.

Figure 2 gives the measured values of Q_A vs δ_V^2 showing a linear relationship, as predicted by expressions (9) and (11), with $y \sim \delta_V^2$. The absolute theoretical value of y depends on the ratio $r = l_s/l_{th}$, as follows from (8) and (11). From [5] it can be found that the flow boundary layer thickness l_f equals $\sqrt{(v/a)}\ l_{th}$ in a periodic boundary layer, where v and a are the kinematic viscosity and the thermal diffusivity, respectively. For helium this ratio is calculated to be 0.75, which means that $l_f/l_{th} = 0.87$. The ratio l_s/l_f will certainly be smaller than 1, due to turbulent mixing in the domain, when $l_s < x < l_f$. Estimating l_s/l_f to be 0.5 (see [6]), we find as the value of $r = l_s/l_{th} = 0.44$. Figure 2 gives the values of Q_A calculated from (8) and (11), with $T_A = T_B = 296$ K, $\lambda_o = 0.15$ W/mK, $a = 2.93\ 10^{-6}\ \mathrm{m}^2/\mathrm{s}$, $l_{th} = 0.147\ 10^{-3}$ m, and $r = 0.44$. It is seen that a reasonable agreement between experimental values and values calculated with $r = 0.44$ is obtained.

For comparison values of Q_A calculated from (8) and (11) with $r \gg 1$, and values calculated with aid of heat transfer correlations given by Nusselt [7] and van Tijen [8] are also given. The latter read

$$\alpha = 1.15\, p^{2/3}\, T^{1/3}(1 + 1.24\, c_m) \tag{13}$$

and

$$\alpha = 1.16\, p^{2/3}\, T^{1/3}(3.19 + 0.885\, c_m), \tag{14}$$

respectively, where p is the gas pressure in bar and c_m is the average piston velocity.* The parameters α is multiplied by the instantaneous surface area and temperature difference between gas and walls in order to obtain Q_A and Q_B by means of a numerical calculation of the heat flows. Figure 2 shows that both latter expressions do not provide the δ_V^2 dependence of Q_A as found experimentally. Apart from this, these values give too low heat flows as compared with the results of the measurements.† It has been calculated that the experimental results can be described by a constant α equal to 2300 W/m^2 K.

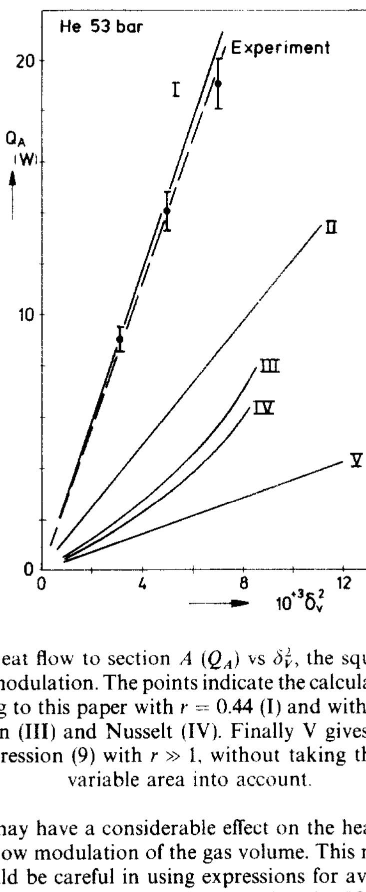

FIG. 2. Heat flow to section A (Q_A) vs δ_V^2, the square of the volume modulation. The points indicate the calculated values according to this paper with $r = 0.44$ (I) and with $r \gg 1$ (II), Van Tijen (III) and Nusselt (IV). Finally V gives the value from expression (9) with $r \gg 1$, without taking the effect of variable area into account.

CONCLUSIONS

From the above preliminary results on heat transfer during piston compression and expansion it can be concluded that variation of the heat transferring surface area with the piston motion may have a considerable effect on the heat transfer, even for low modulation of the gas volume. This means that one should be careful in using expressions for average heat transfer in time dependent systems, as given by Pfriem [4]. It has been found that a simple boundary layer theory, with $l_s/l_{th} = 0.44$ provides a heat transfer coefficient, that is in agreement with the experimental data obtained. Both the above theory with $l_s/l_{th} \gg 1$ and data derived from Nusselt and van Tijen provide too low values for the heat transfer in the present case, the difference however being that the above boundary layer theory provides a correct δ_V-dependence of Q_A, indifferent to the value of l_s/l_{th}.

Future work should include a more detailed study of the time dependent boundary layer, on which more information may be obtained by studying the influence of $\Delta T = T_A - T_B$ on the quantity Q_A [see expression (9)].

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*An extensive review of the various heat transport expressions given in the literature can be found in [9] and [10].

†This demonstrates that the effect of gas flow, basis of the expressions (13) and (14) is much lower than the effect of gas temperature variations, discussed in this paper.

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